Published by Institute of Physics Publishing for SISSA/ISAS
Received: July 6, 2004
Accepted: September 13, 2004

Towards a theory of flavor from orbifold GUTs

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Abstract: We show that the recently constructed 5-dimensional supersymmetric $S^1/(Z_2 \times Z_2')$ orbifold GUT models allow an appealing explanation of the observed hierarchical structure of the quark and lepton masses and mixing angles. Flavor hierarchies arise from the geometrical suppression of some couplings when fields propagate in different numbers of dimensions, or on different fixed branes. Restrictions arising from locality in the extra dimension allow interesting texture zeroes to be easily generated. In addition the detailed nature of the SU(5)-breaking orbifold projections lead to simple theories where $b-\tau$ unification is maintained but similar disfavored SU(5) relations for the lighter generations are naturally avoided. We find that simple 5d models based on $S^1/(Z_2 \times Z_2')$ are strikingly successful in explaining many features of the masses and mixing angles of the 2nd and 3rd generation. Successful three generation models of flavor including neutrinos are constructed by generalizing the $S^1/(Z_2 \times Z_2')$ model to six dimensions. Large angle neutrino mixing is elegantly accommodated. Novel features of these models include a simple $m_u = 0$ configuration leading to a solution of the strong CP problem.

Keywords: Field Theories in Higher Dimensions, Quark Masses and SM Parameters, GUT.

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Contents

1. Introduction

Since the pioneering work of Georgi and Glashow [1] (also [2]) the compelling concept of a grand unified theory (GUT) of the Standard Model (SM) gauge interactions has dominated our thinking about physics at high energies. The quantitative success of gauge coupling unification [3] in the minimal supersymmetric standard model (MSSM) extensions of these theories [4, 5] has provided further support for this idea [6].

These successes motivate one to look for similar high-scale explanations of the rich flavor structure we observe at low energies. While the three Yukawa matrices appear as independent parameters in the Standard Model, they can become correlated in an extension: e.g. , in specific GUTs there are relations between fermion masses, such as the SU(5) relation $m_b = m_\tau$ [7]. As with the unification of gauge couplings this relation applies only at the unification scale and is modified by renormalization group (RG) running down to the weak scale. Indeed, these corrections offer a further tantalising piece of circumstantial evidence in favour of supersymmetric unification for, starting with the relation $m_b(M_X) = m_\tau(M_X)$, they bring the prediction for m_b/m_τ into agreement with experiment using the same value for the unification scale M_X as found in the analysis of gauge couplings. This illustrates both how the analysis of fermion masses can lend support to the hypothesis of (supersymmetric) unification, and how we can learn about the puzzling flavor structure of the SM from unified theories.

However, additional predictions for fermion masses and mixing angles require more sophisticated unified theories. One reason for this is that the simple relations $m_s = m_\mu$ and $m_d = m_e$ are in gross disagreement with experiment even with RG corrections included. Such problems inspired Georgi and Jarlskog [8] to study a different unified ansatz, which was then developed and extended to other ansätze in many subsequent studies with some success (see for example [9, 10]). Despite this, one problem that looms over GUTs, and especially their extensions addressing such flavor structure, is the difficulty of building a Higgs structure that achieves the desired pattern of vacuum expectation values (VEVs) and breaking. Most notorious is the doublet-triplet splitting problem of SU(5) and other GUTs. Although solutions, such as the Dimopoulos-Wilczek mechanism in SO(10) are possible [11], the full Higgs structure necessary to realise the flavor relations tends to be unattractive. In addition the constraints from the non-observation of proton decay are now so severe as to disfavor or even rule out many of the simplest models [12].

Recently, a new possibility for the embedding of the SM into a form of GUT has been suggested in refs. [13]–[19]. The idea is that the GUT gauge symmetry is realized in 5 or more space-time dimensions and only broken down to the SM by compactification on a singular 'orbifold', utilizing GUT-symmetry violating boundary conditions. Given the success of the traditional supersymmetric gauge-coupling unification predictions, the most attractive model is one with both supersymmetry and (in a sense we make precise) SU(5) gauge symmetry. Then in the 5d case both the GUT group and 5d supersymmetry are broken by compactification on $S^1/(Z_2 \times Z_2')$ down to a N=1 SUSY model with SM gauge group. This construction allows one to avoid some unsatisfactory features of conventional GUT's with Higgs breaking, such as doublet-triplet splitting, while maintaining, at least at leading order, the desired gauge coupling unification [15, 16, 21].[1] The significant advantage of the effective field theory approach to orbifolds that we advocate, is that large classes of (consistent low-energy) models can be surveyed, and phenomenologically interesting features or mechanisms identified.

In this paper we examine what we consider a very appealing explanation of the hierarchical structure of the quark and lepton masses and mixing angles based on a generalization of the $S^1/(Z_2 \times Z_2')$ orbifold GUT model. Our hypothesis is that flavor hierarchies are a result of the geometrical suppression of some couplings due to wavefunction normalization factors arising when fields propagate in different numbers of dimensions. Locality and the detailed nature of the orbifold projections in field space play significant roles as fixed branes of various dimensionalities and in various locations are an automatic feature of the $S^1/(Z_2 \times Z_2')$ orbifold mechanism for SU(5) breaking. We find that quite simple models of this sort lead to notably successful flavor textures.

The outline of our paper is as follows. In section 2 we briefly recall the essential features of the $S^1/(Z_2 \times Z_2')$ orbifold mechanism, paying particular attention to aspects that are important in the construction of our theory of flavor. In section 3 we explain the basic idea of our approach to the flavor problem, while section 4 is devoted to the examination of these ideas in the simplified context of models for the flavor structure

[1] These features are shared with those of the original string orbifold constructions [20].

of the heaviest 2 generations. We find that a simple hypothesis is strikingly successful in explaining many features of 2nd and 3rd generation masses and mixing angles: the only matter propagating in the 5th dimension is the second generation **10** of SU(5). Full three generation models of flavor require that we generalize this $S^1/(Z_2 \times Z_2')$ model to six dimensions. The construction and appealing aspects of such models are discussed in section 5. Finally, section 6 contains our conclusions.

2. Basics of the $S^1/Z_2 \times Z_2'$ model

In this Section we provide an introduction to the physics of the simplest orbifold GUT model, the SU(5) model on the 5 dimensional space $M_4 \times (S^1/Z_2 \times Z_2')$. We closely follow the discussion and notation of Refs. [15, 16].

In addition to 4d Minkowski space consider a 5th dimension with orbifold structure $S^1/(Z_2 \times Z_2')$. (We label the usual 4d coordinates x^μ, $\mu = 0, \ldots, 3$, while the 5th coordinate $x^5 \equiv y$.) The circle S^1 is assumed to have radius R where $1/R \sim M_{\rm GUT}$. The orbifold $S^1/(Z_2 \times Z_2')$ is obtained by modding out the theory by two Z_2 transformations which impose on bulk fields the equivalence relations: $P: \ y \sim -y$ and $P': \ y' \sim -y'$ (here $y' \equiv y - \pi R/2$). Under these actions there are two inequivalent fixed 3-branes (or 'orbifold planes'), which we denote O and O', located at $y = 0$, and $y = \pi R/2 \equiv \ell$, respectively. The physical domain of the theory is the interval $y \in [0, \ell]$ with the O, O' branes acting as 'end-of-the-world' branes.

The action of the equivalences P, P' must also be defined within the space of fields: $P: \Phi(x,y) \sim P_\Phi \Phi(x,-y)$ and $P': \Phi(x,y') \sim P'_\Phi \Phi(x,-y')$, where here, P_Φ and P'_Φ are matrix representations of the two Z_2 actions, and we can classify the fields by their (P, P') eigenvalues $(\pm 1, \pm 1)$. Then the KK expansions of bulk fields $\Phi_{PP'}(x,y)$ involve $\cos(ky/R)$ with $k = 2n$ or $2n+1$, for $\Phi_{+P'}$ ($P' = +, -$ respectively), and $\sin(ky/R)$ with $k = 2n+1$ or $2n+2$, for $\Phi_{-P'}$ ($P' = +, -$ respectively). Only the Φ_{++} possess a massless zero mode, the other KK modes acquire a mass k/R from the 4d perspective. Only Φ_{++} and Φ_{+-} have non-zero values at $y = 0$, while only Φ_{++} and Φ_{-+} are non-vanishing at $y = \ell$. The action of the identifications P, P' on the fields (the matrices P_Φ and P'_Φ) can involve any symmetry of the bulk theory; gauge transformations, discrete parity transformations, and R-symmetry transformations in the supersymmetric case; and this allows one to break the bulk symmetries.

The starting point for phenomenology is a 5d SU(5) gauge theory with minimal 5d SUSY (8 real supercharges, corresponding to $N = 2$ SUSY in 4d). Thus, at minimum, the bulk must have the 5d vector superfield, which in terms of 4d $N = 1$ SUSY language contains a vector supermultiplet V with physical components A_μ, λ, and a chiral multiplet Σ with components ψ, σ. Both V and Σ transform in the adjoint representation of SU(5). If the parity assignments, expressed in the fundamental representation of SU(5), are chosen to be $P = \mathrm{diag}(+1,+1,+1,+1,+1)$, and $P' = \mathrm{diag}(-1,-1,-1,+1,+1)$, so that the equivalence under P is

$$V^a(x,y)T^a \sim V^a(x,-y) P T^a P^{-1}\,, \tag{2.1}$$

and similarly for P', then SU(5) is broken to SU(3) $\times$ SU(2) $\times$ U(1) on the O' fixed-brane, but is unbroken in the bulk and on O. If for Σ the same assignments are taken apart from an overall sign for both P and P' equivalences, e.g. , under P,

$$\Sigma^a(x,y)T^a \sim -\Sigma^a(x,-y)PT^aP^{-1}\,, \tag{2.2}$$

then these boundary conditions also break 4d $N=2$ SUSY to 4d $N=1$ SUSY on both the O and O' branes. Only the $(+,+)$ fields V^a possess massless zero modes (a labels the unbroken SU(3) $\times$ SU(2) $\times$ U(1) generators of SU(5), while $\hat{a}$ labels the broken generators), and at low energies only the gauge content of the 4d $N=1$ MSSM is apparent.

On the other hand, the bulk of the theory is invariant under both the full SU(5) gauge symmetry and the full 5d minimal supersymmetry. If we take the MSSM matter to reside in the bulk, then they must come in complete SU(5) multiplets. In fact the correct situation with regard to quantum numbers is slightly more subtle. The reason for this is that the bulk $N=2$ SUSY appears to pose a problem. The minimal matter superfield representation for such a theory is a hypermultiplet, which in 4d N=1 language decomposes in to a chiral multiplet $\Phi_{\mathbf{R}}$ together with a mirror chiral multiplet in the conjugate representation $\Phi^c_{\overline{\mathbf{R}}}$. Thus, the choice of matter in the bulk would appear to have problems reproducing the chiral structure of the SM.

Fortunately as shown in detail in ref. [16] (see also [15]) the structure of the orbifold projections P and P' acting on fields resolves this in a particularly interesting fashion. The action of these projections on the N=1 component fields Φ and Φ^c residing in a 5d hypermultiplet is inherited from the action on the 5d vector multiplet eqs. (2.1)–(2.2). The result is that actions of both P and P' on the 4d chiral fields Φ and Φ^c have a relative sign:

$$P:\Phi \sim P_\Phi \Phi\,, \qquad P:\Phi^c \sim -P_\Phi \Phi^c\,, \tag{2.3}$$

and similarly for P'. This difference leads to a chiral spectrum for the zero modes. Indeed the KK spectrum of 5d bulk hypermultiplets in the representations $\mathbf{10}$ and $\overline{\mathbf{5}}$ (whose 4d chiral components we denote $T+T^c+\overline{F}+\overline{F}^c$) resulting from the P,P' actions is given in table 1. Note that since P' does not commute with SU(5), components in different SU(3) $\times$ SU(2) $\times$ U(1) representations have different KK mode structures. Thus $Q,\overline{U},\overline{D},L,\overline{E}$, etc., are used to indicate their SM transformation properties. It is important in the following that the zero modes under this action *do not fill out full* SU(5) *multiplets* of SM matter. From T we just get $\overline{U}$ and $\overline{E}$ zero modes, while from $\overline{F}$ we get L.

The remaining components of a full SU(5) multiplet are realised at the zero mode level by taking another copy of $\mathbf{10}$ and/or $\overline{\mathbf{5}}$ in the bulk (with $N=1$ chiral components denoted $T'+T'^c$ and $\overline{F'}+\overline{F'}^c$ respectively), and using the freedom to flip the overall action of the P' parity on these multiplets by a sign relative to the action on $T+\overline{F}$. This difference leads to a different selection of zero mode components, the KK spectrum being given in table 2.

Combining the results of tables 1 and 2 we have zero modes which fill out the full matter content of either (or both) a $\mathbf{10}$ or $\overline{\mathbf{5}}$ multiplet of SU(5) at the zero mode level, but without the doubling due to $N=2$ bulk SUSY. Moreover, since different components

(P,P')	4d superfield	4d mass
$(+,+)$	$T_{\overline{U}}, T_{\overline{E}}, \overline{F}_L$	$2n/R$
$(+,-)$	$T_Q, \overline{F}_{\overline{D}}$	$(2n+1)/R$
$(-,+)$	$T^c_{\overline{Q}}, \overline{F}^c_D$	$(2n+1)/R$
$(-,-)$	$T^c_U, T^c_E, \overline{F}^c_{\overline{L}}$	$(2n+2)/R$

Table 1: Parity assignments and KK masses of fields in the 4d chiral supermultiplets resulting from the decomposition of 5d hypermultiplets in the $(T+\overline{F})$ representation. The subscript labels the SM transformation properties, e.g. , $Q = (\mathbf{3},\mathbf{2})_{\mathbf{1/6}}$, $\overline{Q} = (\overline{\mathbf{3}},\overline{\mathbf{2}})_{\mathbf{-1/6}}$, $\overline{U} = (\overline{\mathbf{3}},\mathbf{1})_{\mathbf{-2/3}}$, etc.

(P,P')	4d superfield	4d mass
$(+,+)$	$T'_Q, \overline{F'}_{\overline{D}}$	$2n/R$
$(+,-)$	$T'_{\overline{U}}, T'_{\overline{E}}, \overline{F'}_L$	$(2n+1)/R$
$(-,+)$	$T'^c_U, T'^c_E, \overline{F'}^c_{\overline{L}}$	$(2n+1)/R$
$(-,-)$	$T'^c_{\overline{Q}}, \overline{F'}^c_D$	$(2n+2)/R$

Table 2: Parity assignments and KK masses of fields in 4d chiral supermultiplets resulting from the decomposition of 5d hypermultiplets in the $(T'+\overline{F'})$ representation.

of what we think of as a single **10** or $\overline{\mathbf{5}}$ in fact arise from different parent multiplets in the higher dimension, the usual SU(5) counting of independent couplings, and thus the relations between masses and mixing angles is modified. We will soon utilize this feature in the construction of realistic models of flavor.

3. Flavor hierarchies from geometry

First, to set notation, recall that in the Standard Model, the quark and lepton masses and the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix are parameterized in terms of three 3×3 matrices, (U,D,E) of Yukawa coupling constants. In the MSSM, and after electroweak symmetry breaking the Yukawa interactions lead to:

$$\mathcal{L}_{\rm mass} = \overline{Q}^i_L U_{ij} u^j_R \frac{v}{\sqrt{2}} \sin\beta + \overline{Q}^i_L D_{ij} d^j_R \frac{v}{\sqrt{2}} \cos\beta + \overline{L}^i_L E_{ij} e^j_R \frac{v}{\sqrt{2}} \cos\beta + \text{h.c.}\,, \tag{3.1}$$

where $i,j=1,2,3$ are generation indices, $\tan\beta = \langle H_u\rangle / \langle H_d\rangle$, is the ratio of Higgs vacuum expectation values, and $v = 246\,\text{GeV}$. The full Yukawa matrices, U and D are not determined by experiments, rather they fix only the diagonal mass matrices, $U^{\rm diag}$, and $D^{\rm diag}$, and the CKM mixing matrix, V_{CKM}. The relations between these objects are specified by the diagonalizing unitary rotations L_u, R_u, L_d, and R_d acting in generation space on the up and down sectors:

$$U^{\rm diag} = L_u U R_u^\dagger\,, \qquad D^{\rm diag} = L_d D R_d^\dagger\,, \qquad V_{CKM} = L_u L_d^\dagger\,. \tag{3.2}$$

Our basic strategy to go beyond the usual SU(5) treatment of the flavor structure of the SM matter is to use the geometrical suppression of some couplings due to the wave-function normalization factors that arise when some fields propagate in different numbers of dimensions (this is similar to the arguments used in the context of neutrinos and other SM neutral states in the case of large extra dimensions [25]).

An illustrative example of the physics is provided by the following slightly simplified model: Consider the 5th dimensional line segment $y \in [0, \pi R/2]$ of the orbifold models with SU(5)-invariant O-brane at $y = 0$, and SU(5) breaking O' brane at $y = \ell \equiv \pi R/2$. There are then three possible locations for each multiplet: in the SU(5)-preserving 5d bulk, or at the O or O' branes. If the light SM states arise from the zero modes Φ_0 of a bulk field (generically denoted Φ) then the normalization factor of these zero modes is $1/\sqrt{M_* \ell}$, where M_* is the UV cutoff scale of our effective higher-dimensional theory.[2] Alternatively some SM states can come from brane localized fields — generically ψ — which we initially take to be located at $y = 0$ for simplicity. Then the Yukawa interactions in the 4d effective lagrangian are of the form (x dependence suppressed):

$$\begin{aligned}\mathcal{L} &\simeq \int dy \left\{ \lambda_0 \delta(y) \psi^3 + \lambda_1 \delta(y) \psi^2 \Phi(y) + \lambda_2 \delta(y) \psi \Phi^2(y) + \lambda_3 \delta(y) \Phi^3(y) + \lambda_3' \Phi^3(y) \right\} \\ &\simeq \lambda_0 \psi^3 + \frac{\lambda_1}{(M_* \ell)^{1/2}} \psi^2 \Phi_{(0)} + \frac{\lambda_2}{(M_* \ell)} \psi \Phi_{(0)}^2 + \frac{\lambda_3}{(M_* \ell)^{3/2}} \Phi_{(0)}^3 + \frac{\lambda_1'}{(M_* \ell)^{1/2}} \Phi_{(0)}^3 \,. \end{aligned} \tag{3.3}$$

This equation, which is crucial for the rest of our analysis, requires some explanation. For any interaction which contains at least one brane localised field (λ_0, λ_1, or λ_2) there is automatically a localising $\delta(y)$-function when the lagrangian is written in 5d. Then, for these terms, the number of factors of $1/\sqrt{\ell}$ in the effective 4d lagrangian is simply given by the number of bulk fields in the interaction, as each one has this wavefunction factor. However for an interaction that involves only bulk fields Φ there are two possibilities. The first (λ_3') is that the Yukawa interaction occurs *everywhere* in the bulk. In this case there is no δ-function, and for a cubic term two of the wavefunction factors get cancelled by the integral over $y \in [0, \ell]$, leading to just one power of $1/\sqrt{M_* \ell}$ overall. The second possibility is that although the fields Φ live in the bulk, the *interaction occurs only on the brane*. This corresponds to the δ-function in the λ_3 coupling, leading to three factors of $1/\sqrt{M_* \ell}$ in the 4d effective coupling. (In the case with more than one brane such an interaction can occur at either, or both of the branes. This can be important in some circumstances.)

The second situation may seem unlikely, however it is in fact the typical case. The reason for this is that, as we mentioned above, the minimal supersymmetry in 5d corresponds to $N = 2$ from the 4d perspective. But $N = 2$ supersymmetry does not allow Yukawa interactions between matter hypermultiplets, so the interactions between the component $N = 1$ chiral multiplets are confined to the branes where the $N = 2$ SUSY is broken to $N = 1$ by the orbifold conditions.

The various possible flavor models are characterized by the placement of the T_i and $\overline{F}_i$ multiplets that make up the three generations of quarks and leptons. In addition it is also possible for the Higgs multiplets to be located in either the bulk or on the O'-brane — if they were on the O-brane the doublet-triplet splitting problem re-emerges. To have at least some SU(5) mass relations, such as $b - \tau$ unification, the Higgs must reside in the SU(5)-symmetric bulk, in which case they form components of $\mathbf{5}$ and $\overline{\mathbf{5}}$ in the bulk which we denote H and $\overline{H}$ respectively; for definiteness we will focus on this case in the

[2]The factor of M_* arises from correctly normalizing the kinetic terms so that the zero modes are fields with 4d canonical dimensions.

following. In the next two sections we explore forms for the Yukawa matrices following from the preceding considerations of locality, geometry, and the spatially dependent gauge and supersymmetries. We will not impose extra constraints, such as discrete or continuous flavor symmetries.

4. Two-generation models

For the charged fermion masses of the heaviest two generations there are six dimensionless ratios: $m_\mu/m_\tau \simeq 1/17, m_s/m_b \simeq 1/40, m_c/m_t \simeq 1/300, V_{cb} \simeq 1/25, m_b/m_\tau \simeq 1, m_b/m_t \simeq 1/60$. We aim to provide a qualitative understanding of the first four of these in terms of the single volume factor $\varepsilon \equiv 1/\sqrt{M_* \ell}$, according to $m_\mu/m_\tau, m_s/m_b, V_{cb} \approx \varepsilon$ and $m_c/m_t \approx \varepsilon^2$. In addition, we begin with requiring that m_b/m_τ be precisely understood by the SU(5) mass relation, and explore the most general forms of the Yukawa matrices allowed by locality, without imposing extra constraints. Later we will consider the possibility of relaxing the requirement of precision $b-\tau$ unification. In section 5 we will also show the ratio $m_b/m_t \simeq 1/60$ can be understood from geometry.

The up quark mass matrix in SU(5) is proportional to the coefficient of an operator bilinear in the T_i fields: $T_i T_j$. A hierarchy of eigenvalues requires that one T is placed on a brane and the other in the bulk. The brane localized field has the heaviest up-type quark, and is therefore to be identified with the third generation T_3. To have a hope of obtaining the SU(5) relation $m_b = m_\tau$ at the unification scale, T_3 must be on O rather than O', so that the placement of the two T_i fields is unique. The up quark Yukawa coupling matrix for the two heavy generations, in the "locality basis", is

$$U \simeq \begin{pmatrix} \delta^3 & \delta^2 \\ \delta^2 & \varepsilon \end{pmatrix}. \tag{4.1}$$

In this expression δ corresponds to an SU(5)-violating parameter of the same magnitude as the SU(5)-invariant parameter ε. We emphasize that here and below we display only the hierarchical nature of the Yukawa matrices, ignoring the order unity Yukawa couplings of the 5d theory. We conclude that m_c/m_t is necessarily of order ε^2, and that V_{cb} necessarily contains a piece of order ε from diagonalization of the up quark sector.

One interesting feature of eq. (4.1) is that because of the SU(5) violation U is not symmetric as it would be in a 4d SU(5) theory. Also note that the top quark Yukawa coupling has a magnitude proportional to ε. This is not problematic, but shows that the higher dimensional Yukawa coupling is closer to strong coupling than in 4d theories.

The operators leading to down quark and charged lepton mass matrices have the form $T_i \overline{F}_j$, leading to the 4d Yukawa matrices D_{ij} and E_{ji}. These matrices will have a hierarchy of order ε on the i index due to the locality of the T_i. Since we require mass ratios $m_\mu/m_\tau, m_s/m_b \approx \varepsilon$, this implies that there is no additional hierarchy between the two generations resulting from the index j: $\overline{F}_2$ and $\overline{F}_3$ must be located on branes of the same dimensionality. Indeed it is well-known that the large hierarchy in the up quark sector suggests that, in SU(5) theories, the hierarchy is somehow associated with the T_i rather than with the $\overline{F}_j$. In our scheme this difference has a simple geometrical origin: the T_i reside on branes with a hierarchy of volumes, while the $\overline{F}_j$ do not.

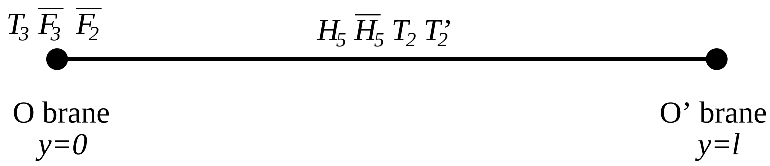

Figure 1: The location of the Higgs and 2nd and 3rd generation matter multiplets. The only alternative is for $\overline{F}_2$ to be located at O' instead of O.

For locality to give the SU(5) m_b/m_τ mass relation, $\overline{F}_3$ must be located on the SU(5) brane O. If it is in the bulk, then b and τ arise from different SU(5) multiplets $\overline{F}_3$ and $\overline{F}'_3$, so that they have unrelated Yukawa couplings. Hence we are forced to conclude that the b and t Yukawa couplings are both of order ε. Thus in this 5d scheme the large t/b mass ratio must arise from a large value for $\tan\beta$, the ratio of electroweak VEVs, rather than from volume factors. (At the end of this section we discuss models without precision $b-\tau$ unification where the large m_t/m_b mass ratio is also explained by geometry.) Thus our two generation theory is as shown in figure 1; the only lack of uniqueness in the choice of location for the second and third generations is whether $\overline{F}_2$ resides at O or O'.

With $\overline{F}_{2,3}$ both on O, we may relabel the combination which couples to T_3 as $\overline{F}_3$, giving the Yukawa couplings in the "locality basis":

$$D \simeq \begin{pmatrix} \delta^2 & \delta^2 \\ 0 & \varepsilon \end{pmatrix}, \qquad E \simeq \begin{pmatrix} \delta^2 & 0 \\ \delta^2 & \varepsilon \end{pmatrix}. \tag{4.2}$$

The weak mixing V_{cb} also acquires a contribution of order ε from diagonalization of the down quark sector. These order ε contributions to quark flavor mixing, from both up and down sectors, are simply a reflection of having T_2 in the bulk and T_3 on the brane. Eqs.(4.1) and (4.2) together lead to

$$\frac{m_\mu}{m_\tau} \simeq \frac{m_s}{m_b} \simeq V_{cb} \simeq \varepsilon\,, \qquad \frac{m_c}{m_t} \simeq \varepsilon^2\,, \qquad \text{and} \qquad m_b = m_\tau\,, \tag{4.3}$$

as desired. Indeed, it is remarkable, as can be seen from figure 1, that so much of the flavor structure of the heaviest two generations is so simply understood by the single device of putting T_2 in the bulk. Finally note that there is no change to these results even if $\overline{F}_2$ is located at O'. The operator $T_3\overline{F}_2\overline{H}$ is now forbidden by locality, so that the Yukawa matrices again take the form of eq. (4.2).

Before we discuss neutrino masses, let us mention an interesting variation. While the present configuration gives $m_b = m_\tau$, it requires a large $\tan\beta$, as m_b/m_t is not volume-suppressed. However, it is possible to get the right m_b/m_t without a large $\tan\beta$, by putting both $\overline{F}_2$ and $\overline{F}_3$ in the bulk. Since the locations of T's are unaltered, the up-quark mass matrix eq. (4.1) stays the same, while the down-quark and lepton matrices in eq. (4.2) become smaller by a factor of ε, which is good for m_b/m_t. However, the precise SU(5) relation $m_b = m_\tau$ is lost. We will see how both can be achieved in a 6d model.

4.1 Neutrino masses

Neutrino masses can be generated via the see-saw mechanism if SU(5) singlet fields N are introduced. If the N's are located at a 3-brane, they are expected to acquire Majorana mass of order M_*. For bulk N's, contributions to Majorana masses from brane mass terms are of order $1/\ell$. In addition to this, we can write down bulk mass terms if we have more than one hypermultiplet in the bulk, which gives masses of order M_*, but here, for simplicity, we assume that these large bulk masses are absent.

Now, as an interesting example, consider the case of a single N field. With $\overline{F}_2$ at O, large $\nu_\mu \nu_\tau$ mixing is inevitable: $V_{\mu\tau} \approx 1$. The $\overline{F}_{2,3}$ basis is defined by the couplings of T_3, so that the two couplings $\overline{F}_{2,3}NH$ will be comparable independent of where N is located. This is a striking result: so much of the observed pattern of flavor in the heavy two generations appear to emerge from putting all the matter on the brane with SU(5) invariance, except for the field T_2.[3]

It would be exciting to obtain a neutrino mass enhanced from v^2/M_G by an inverse power of ε, corresponding to the measured atmospheric oscillation distance, but this does not seem to readily occur. One typically gets a suppression: $\varepsilon^2 v^2/M_G$ for bulk N, or v^2/M_* for brane N. Therefore it seems that an additional ingredient is necessary to suppress the Majorana NN mass. For example, with $\overline{F}_3$ and N both located on O, the NN mass might be suppressed if $(B-L)$ breaking occurs at O' and is mediated across the bulk by heavy fields. If a second N field is placed in the bulk it would have a much larger Majorana mass, and would give rise to a correspondingly smaller light neutrino eigenvalue. On extending the theory to three generations, this eigenvalue can provide the smaller neutrino mass squared difference required for solar neutrino oscillations.

5. Three-generation models in six dimensions

Attractive three generation flavor models where all large hierarchies are explained by geometry can be constructed using a simple extension of the $S^1/(Z_2 \times Z_2')$ theories to 6 dimensions. Specifically consider a 2d "rectangular" extra dimensional space with coordinates y_1, y_2 formed by the product of two $S^1/(Z_2 \times Z_2')$ structures at right angles. Equivalently this is just a 2d torus T^2 modded out by $(Z_2 \times Z_2')^2$. This rectangle has 4 orbifold 4-branes at its border, O_1 along the y_1 axis, O_2 along the y_2 axis, O_1' along $y_2 = \ell_2$, and O_2' along $y_1 = \ell_1$. We take $\ell_2 > \ell_1$.

This 6d system has a richer set of possible symmetry structures than the 5d case case. In particular the two sets of orbifold actions, $(Z_2 \times Z_2')_1$ acting in the y_1 direction and $(Z_2 \times Z_2')_2$ acting in the y_2 direction, can have different actions in field space:

A. The most immediate extension of the 5d models is if for each factor, Z_2 commutes with SU(5) and Z_2' commutes with only SU(3) $\times$ SU(2) $\times$ U(1). In this case the O_1

[3] On the other hand, if we were to consider models with $\overline{F}_2$ and $\overline{F}_3$ spatially separated, both small and large mixing angle possibilities exist. With N in the bulk $V_{\mu\tau} \approx 1$, while with N on a brane $V_{\mu\tau} \approx \varepsilon^2$. One's naive expectation that quark-lepton unification will give $V_{\mu\tau} \approx V_{cb} \approx \varepsilon$ turns out not to be correct. There is no relation between the mixing of quark doublets (contained in T) and lepton doublets (contained in $\overline{F}$) because SU(5) allows a different spatial arrangement for the T and $\overline{F}$ fields.

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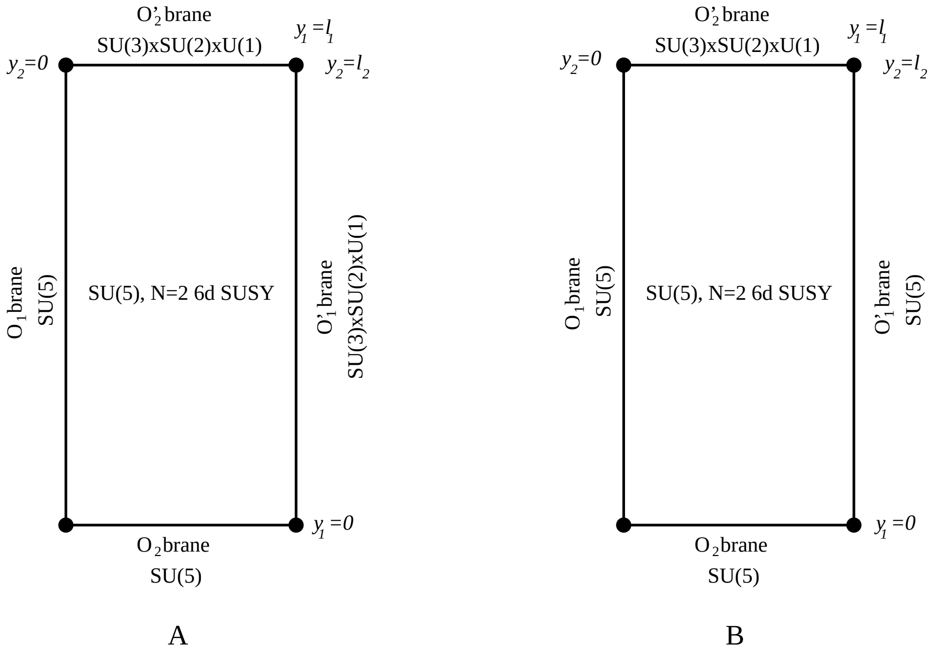

Figure 2: Two of the possible structures of gauge and SUSY breaking in 6d generalized from the $S^1/(Z_2 \times Z_2')$ model in 5d. In both cases all 5d boundary branes possess $N = 1$ 5d SUSY, while all 4d fixed-brane corners have only 4d $N = 1$ SUSY.

and O_2 branes preserve SU(5) with (from a 4d counting) $N = 2$ SUSY, while the O_1' and O_2' branes preserve just $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. This symmetry structure is illustrated in figure 2A.

B. Other options arise if we reduce the amount of SU(5) breaking on the orbifold. If instead of equivalent actions in the two directions we change the gauge properties of one of the factors so that both Z_2 and Z_2' commute with SU(5), we can have a situation where each 4-brane is SU(5) preserving except O_2', say, which only preserves $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. This is illustrated in figure 2B.

Note that in both cases the bulk possesses the full SU(5) gauge symmetry, but now with $N = 2$ SUSY from the 6d perspective, equivalent to $N = 4$ in 4d. This ensures that the 6d bulk theory is free of gauge anomalies [17, 19]. The corners of the rectangle are 3-branes which we will identify with our 4-dimensional world. The combination of orbifold projections at these corners breaks the 6d $N = 2$ SUSY all the way down to $N = 1$ 4d supersymmetry.

Another new feature of the 6d case is the possibility of placing in different locations the fields giving rise to zero modes that effectively fill out complete SU(5) multiplets. For example, in the $S^1/(Z_2 \times Z_2')$ orbifold of section 4 we needed to have both T_2 and T_2' fields to realize zero modes of matter in a full **10** of SU(5), and moreover, because there was only one extended 5d space (the full bulk in this case) they necessarily had to be located together. However in both 6d cases illustrated in figure 2 there are now 4 fixed branes of spacetime dimensionality 5 on which we can place the matter and explain hierarchies by volume factors. In some cases, such as the branes O_1 and O_2 of structure A, or O_1 and O'_1 of structure B, the full SU(5) symmetry is realized on both branes. In such cases we are allowed to place, e.g. , for structure B, T_1 on O_1, while T'_1 is located on O'_1. As we will discuss below this allows us to engineer some interesting "texture zeros". Alternatively we could utilize matter situated on the branes that do not preserve SU(5), but only the SM subgroup. In this situation [16] it is only necessary that SM multiplets are localized to the brane, and there is nothing that forces, say, both the $\overline{U}$ and $\overline{E}$ components of what was formerly combined in a T to be placed on the same brane, or, as we will see, $\overline{U}_1$ to be present at all if we wish to have a massless up-quark. (It is also amusing to note that such constructions in orbifold GUT theories allow exotic states with leptoquark quantum numbers in a manner that is naturally consistent with the stringent proton decay and flavor-changing constraints [26].)

One worry concerning these 6d models is that the cutoff M_* might not be sufficiently far above the scales $1/\ell_{1,2}$ if the 6d gauge theory quickly becomes strongly coupled. If this were the case then we would not be able to take the parameters $\varepsilon_1 = 1/\sqrt{M_*\ell_1}$ and $\varepsilon_2 = 1/\sqrt{M_*\ell_2}$ sufficiently small to be useful in generating the observed flavor hierarchies. However the usual naive dimensional analysis or RG running estimate of the cutoff scale is very dependent on the flat nature of the extra dimensions (with the usual evenly spaced Kaluza-Klein spectrum). As shown in ref. [27] the form of the KK spectrum is highly dependent on the curvature of the extra dimensional space (apart from the zero modes which are unaffected since they arise for essentially topological reasons). For example, replacing the $g = 1$ torus T^2 with a simple 2d compact hyperbolic manifold (CHM) (genus $g > 1$ Riemann surfaces) leads to an exponential squeezing of the excited KK modes to high scales. (Specifically, the non-zero-mode KK states have masses that start at $\exp(\alpha)/\ell$ rather than $m_{KK} \sim 1/\ell$, for some α which depends on the CHM chosen and which can easily be $O(10)$.) On the other hand, it is simple to prove that the shape of the zero-mode wavefunctions in the extra dimensions is still constant, and the normalization factors for the zero-mode wavefunctions are still given by $1/\sqrt{M_*\ell}$. The raising of the excited KK spectrum raises the scale at which the 6d gauge theory becomes strong, and so allows larger values of $M_*\ell$.

With these general considerations in mind we now turn to the construction of a variety of three-generation flavor models in 6d. As well as the building of an attractive model our interest is to demonstrate some of the possibilities of the new framework.

5.1 Two simple 3-generation models with small $\tan\beta$

One simple model based on the symmetry structure A of figure 2 is depicted in figure 3. As before, the quark and lepton mass hierarchies arise from the T's: all three generations $\overline{F}_1, \overline{F}_2, \overline{F}_3$ of matter $\overline{\mathbf{5}}$'s are located at the 4D intersection of O_1 and O_2, together with the third generation T_3, ensuring bottom-tau unification. On the other hand, $T_1 + T'_1$ and $T_2 + T'_2$ are located on the 5D spaces O_1 and O_2, respectively. Finally, $\overline{H}_5$ and H_5, containing the down-type and up-type Higgs multiplets, also propagate on O_1 and O_2 respectively. This configuration gives rise to the following textures for Yukawa couplings

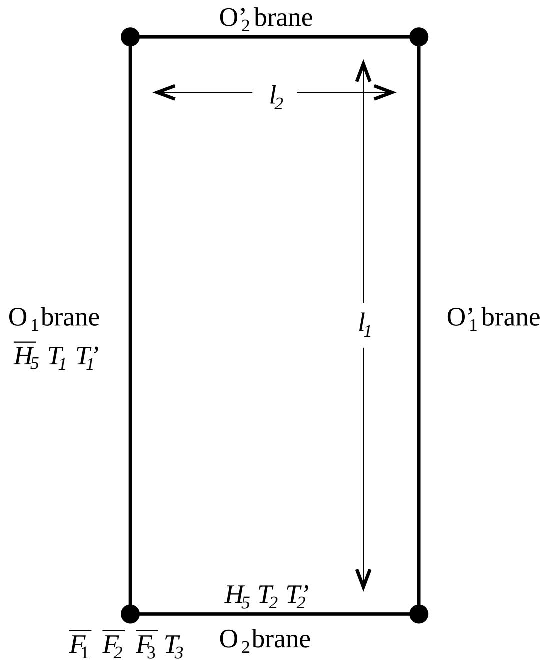

Figure 3: The configuration of the MSSM Higgs and 3 generations of matter multiplets in a simple 6D flavor model based on orbifold structure A.

in the locality basis:

$$U \simeq \varepsilon_2 \begin{pmatrix} \delta_1^2 & \delta_1\delta_2 & \delta_1 \\ \delta_1\delta_2 & \delta_2^2 & \delta_2 \\ \delta_1 & \delta_2 & 1 \end{pmatrix}, \qquad D \simeq \varepsilon_1 \begin{pmatrix} \delta_1 & \delta_1 & \delta_1 \\ 0 & \delta_2 & \delta_2 \\ 0 & 0 & 1 \end{pmatrix}, \qquad E \simeq \varepsilon_1 \begin{pmatrix} \delta_1 & 0 & 0 \\ \delta_1 & \delta_2 & 0 \\ \delta_1 & \delta_2 & 1 \end{pmatrix}. \tag{5.1}$$

Here $(\delta_1, \varepsilon_1)$ and $(\delta_2, \varepsilon_2)$ are volume suppressions of order $1/\sqrt{M_* \ell_1}$ and $1/\sqrt{M_* \ell_2}$ associated with the spaces O_1 and O_2, respectively. As before, the ε's and the 1's in eq. (5.1) are SU(5) preserving while the δ's are SU(5) violating. Also, we have relabelled $\overline{F}_1$, $\overline{F}_2$ and $\overline{F}_3$ such that T_3 couples only to $\overline{F}_3$ while T_2 couples to $\overline{F}_2$ and $\overline{F}_3$. Now, by taking $\ell_1 \gg \ell_2$, we obtain a mass hierarchy for the light two generations, with the hierarchy again strongest for the up-type quarks. Altogether we have $m_t : m_c : m_u \sim 1 : \delta_2^2 : \delta_1^2$ and $m_{b,\tau} : m_{s,\mu} : m_{d,e} \sim 1 : \delta_2 : \delta_1$. The placement of the Higgs multiplets also introduces the hierarchy $\lambda_b/\lambda_t \sim \delta_1/\delta_2$, so in this model m_b/m_t is volume suppressed and large $\tan\beta$ is not necessary. Also note that we do not have the phenomenologically disfavored SU(5) mass relations like $m_d/m_s = m_e/m_u$, because the light generations propagate on 4-branes and are sensitive to the orbifold breaking of SU(5). Thus even this quite simple model has many attractive features.

Much of the discussion on neutrino masses from section 4 also carries over here. Because the $\overline{F}$'s share the same location, large mixing angles are expected regardless of where the N's propagate. (The small value of $\theta_{13} < .15$ [23] should in this model arise as a mild accidental cancellation.) If each N propagates in the same space, this setup provides a simple realization of the neutrino mass anarchy scenario [24]. As before, the Majorana mass for an N localized to the 3-brane may be suppressed by distant breaking of $(B-L)$,

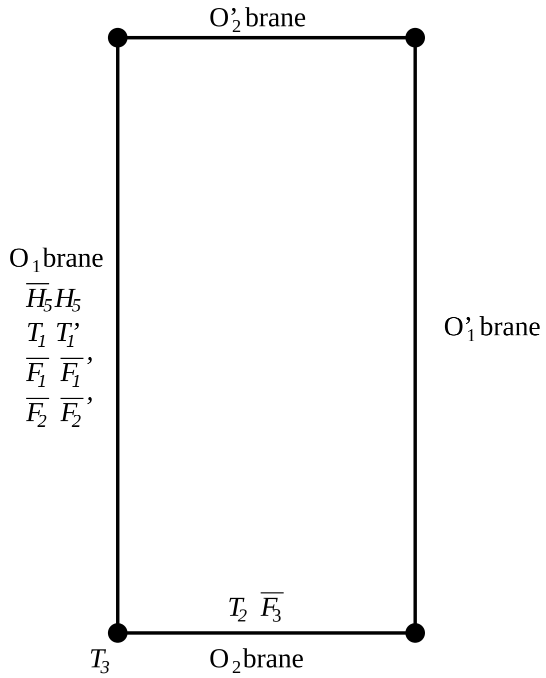

Figure 4: The configuration of the MSSM Higgs and 3 generations of matter multiplets in a simple 6d flavor model based on orbifold structure B.

and if a different N propagating in the bulk or on a 4-brane has direct contact with the breaking, it will have a larger Majorana mass, leading to a smaller mass eigenvalue that could be relevant for solar neutrino oscillations.

To illustrate how the gauge symmetry structure of the orbifold can impact flavor model building, consider another simple model, this time based on structure B (see figure 4). Since the O_2 brane now feels no SU(5) breaking from orbifold projections at either of its endpoints, it is no longer necessary to include the fields T_2' and $\overline{F}_3'$ so as to give the required SM state. As a consequence this construction gives another method of realizing bottom-tau unification even though $\overline{F}_3$ propagates on a 4-brane, as there is just one $\bar{\mathbf{5}}$ state, and thus one Yukawa coupling for both b and τ. Unwanted SU(5) mass relations are still avoided for the light two generations because $\overline{F}_1$ and $\overline{F}_2$ contact the 4-brane O_2' that only preserves SU(3) $\times$ SU(2) $\times$ U(1). By the same token, the Higgs must be located on the vertical line to get doublet-triplet splitting. As before, m_b/m_t is suppressed by a volume factor. The resulting mass matrices, after appropriately relabelling $\overline{F}_1$ and $\overline{F}_2$, are

$$U \simeq \varepsilon_1 \begin{pmatrix} \delta_1^2 & \delta_1\varepsilon_2 & \delta_1 \\ \delta_1\varepsilon_2 & \varepsilon_2^2 & \varepsilon_2 \\ \delta_1 & \varepsilon_2 & 1 \end{pmatrix}, \qquad D \simeq E^T \simeq \varepsilon_1 \begin{pmatrix} \delta_1^2 & \delta_1^2 & \delta_1\varepsilon_2 \\ 0 & \delta_1\varepsilon_2 & \varepsilon_2^2 \\ \delta_1 & \delta_1 & \varepsilon_2 \end{pmatrix}. \tag{5.2}$$

Thus, this model is as attractive as the previous one.

These two models are not perfect, however. They lead to $\theta_c \sim m_d/m_s \sim m_e/m_u \sim \delta_2/\delta_1$, when the correct numerical values are $\sim$ 1/5, 1/20, and 1/200, respectively. Of course because there are unknown coefficients contained in each element of the Yukawa matrices it is not impossible that these ratios are corrected, but this deviation from our

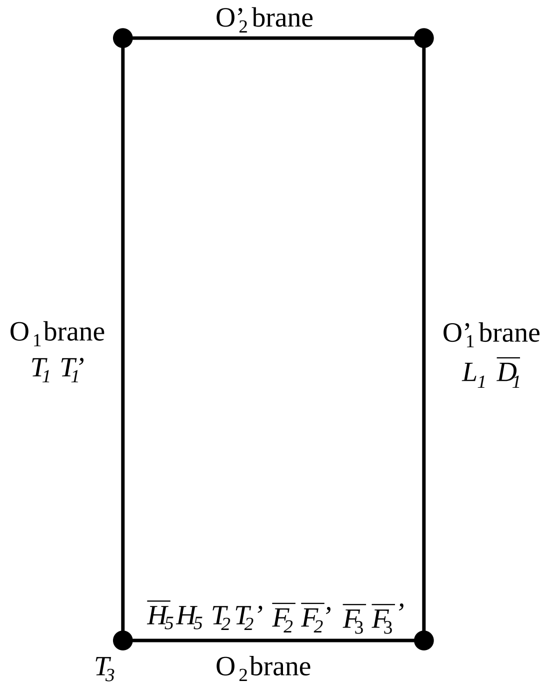

Figure 5: A configuration based on orbifold structure A that gives $D_{11} = E_{11} = 0$, leading to $\theta_c \sim \sqrt{m_d/m_s}$.

philosophy is unappealing. Nevertheless, the textures of eqs. (5.1) and (5.2) do go a long way towards explaining hierarchies of fermion masses and mixings. We now show it is possible to build on this basic idea to do better.

5.2 Improved 3-generation models

Consider a variation of the previous A-type model, shown in figure 5. This demonstrates how texture zeros can arise in our framework (without giving a massless fermion), and also shows how one can improve on some of the mass and mixing-angle relations obtained from eq. (5.1) in the original case. The idea of this model follows from the well-known fact that for the light generations of down-type quarks, the texture

$$D \simeq \begin{pmatrix} 0 & \delta \\ \delta & 1 \end{pmatrix}, \tag{5.3}$$

leads to the successful relation $\theta_c \sim \sqrt{m_d/m_s}$ rather than $\theta_c \sim m_d/m_s$ (provided that larger mixing does not come from the up sector). Moreover, if the E Yukawa texture has the same form, except that its (2,2) entry happens to be somewhat larger, then the texture zero leads to a suppression of the ratio m_e/m_μ relative to m_d/m_s by $(D_{22}/E_{22})^2$. So if the above form of the Yukawa matrix is generated for D and E, the appropriate ratios of θ_c, m_d/m_s, and m_e/m_u can arise merely from a factor of three difference between D_{22} and E_{22} [8] which is acceptable in our framework.

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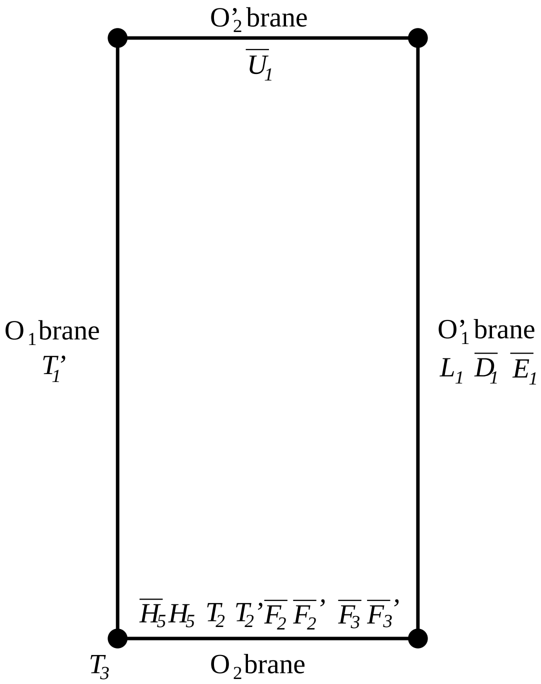

Figure 6: A configuration based on orbifold structure A that gives $\theta_c \sim \sqrt{m_d/m_s}$ and $m_u = 0$.

The configuration of figure 5 realizes this texture by spatially separating T_1 and $\overline{F}_1$, giving the Yukawa matrices

$$U \simeq \varepsilon_2 \begin{pmatrix} \delta_1^2 & \delta_1\delta_2 & \delta_1 \\ \delta_1\delta_2 & \delta_2^2 & \delta_2 \\ \delta_1 & \delta_2 & 1 \end{pmatrix}, \qquad D \simeq E^T \simeq \varepsilon_2 \begin{pmatrix} 0 & \delta_1\delta_2 & \delta_1\delta_2 \\ \delta_2\delta_1 & \delta_2^2 & \delta_2^2 \\ 0 & 0 & \delta_2 \end{pmatrix}, \tag{5.4}$$

in the locality basis. Note that the $(1,1)$ and $(3,1)$ entries of D and E^T vanish because T_1 and T_3 are localized away from L_1 and $\overline{D}_1$. On the other hand the zero in the $(3,2)$ entry of D and E^T is just due to the freedom to relabel the combination of $\overline{F}_2$ and $\overline{F}_3$ that couples to T_3. Thus this model predicts the desirable relation $\theta_c \sim \sqrt{m_d/m_s}$. Another nice feature of this model is that m_b/m_t is suppressed by a volume factor since $\overline{F}_3'$ lives on the O_2 4-brane. However, precision bottom-tau unification is an accident and the model also predicts $m_u/m_c \sim m_d/m_s$ when in fact m_d/m_s is at least a factor ~ 10 larger.

This last difficulty is avoided in an interesting way by the model of figure 6. We have removed T_1 and distributed $\overline{U}_1$ and $\overline{E}_1$ onto O_2' and O_1' respectively. Recall form Tables 1 and 2 that each T on an SU(5) preserving 4-brane (with at least one SU(5)-violating boundary brane) contains massless zero modes only for $\overline{U}$ and $\overline{E}$, while each such T' contains a massless zero mode only for Q, and recall also that O_1' and O_2' preserve just $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. The associated Yukawa matrices are thus

$$U \simeq \varepsilon_2 \begin{pmatrix} 0 & \delta_1\delta_2 & \delta_1 \\ 0 & \delta_2^2 & \delta_2 \\ 0 & \delta_2 & 1 \end{pmatrix}, \qquad D \simeq E^T \simeq \varepsilon_2 \begin{pmatrix} 0 & \delta_1\delta_2 & \delta_1\delta_2 \\ \delta_1\delta_2 & \delta_2^2 & \delta_2^2 \\ 0 & 0 & \delta_2 \end{pmatrix}, \tag{5.5}$$

in the locality basis. The most striking and interesting feature of this model is that it realizes in a simple way two features, $m_u = 0$ and $\theta_c \sim \sqrt{m_d/m_s}$. This is especially so because $m_u = 0$ is a solution to the strong CP problem (as $\overline{U}_1$ has no Yukawa interactions), which may be consistent with chiral perturbation theory [22]. Moreover, the hierarchy m_t/m_b is still explained by volume suppression in this model, again because $\overline{F}'_3$ propagates on a 4-brane. Therefore, except for the fact that precision bottom-tau unification must still be regarded as a numerical accident, this is a notably successful model.

6. Conclusions

We have argued that 5 and 6-dimensional supersymmetric orbifold GUT models allow an appealing explanation of the observed hierarchical structure of the quark and lepton masses and mixing angles. Our hypothesis is that flavor hierarchies arise from the geometrical suppression of some couplings due to wavefunction normalization factors when fields propagate in different numbers of dimensions. Moreover, if fields propagate on different fixed branes, restrictions arising from locality in the extra dimension allow interesting texture zeroes to be simply explained. In addition the detailed nature of the SU(5)-breaking orbifold projections lead to simple theories where $b - \tau$ unification is maintained but similar, disfavored, SU(5) relations for the lighter generations are naturally avoided. We find that simple 5d models based on $S^1/(Z_2 \times Z'_2)$ are strikingly successful in explaining many features of the masses and mixing angles of the 2nd and 3rd generation, this success resulting from the single simple assumption that the only matter propagating in the 5th dimension is the second generation **10** of SU(5). These ideas were then extended in section 5 to successful three generation models of flavor, constructed by generalizing the $S^1/(Z_2 \times Z'_2)$ model to six dimensions on orbifolds with structure $T^2/(Z_2 \times Z'_2)^2$. Once again the primary hypothesis leading to attractive models was the distribution of the three generations of **10**'s, T_1, T_2, T_3 on branes of different co-dimensionality or linear extent in the extra dimensions. Some novel features of these models, including a simple $m_u = 0$ configuration leading to a solution of the strong CP problem were also discussed.

Finally, a valid criticism of our models is that they only give the hierarchical structure of the quark and lepton masses and mixing angles and do not allow precise predictions because of the many unknown $O(1)$ Yukawa couplings of the high-scale theory. In this regard our models share features of the Froggert-Nielsen mechanism [9], where such uncertainties are also present. We believe that by utilizing further features unique to higher-dimensional GUT theories it is possible to fix many of these $O(1)$ parameters, thus leading to a set of precise predictions for relations among the low-energy quark and lepton masses and mixing angles. Certainly many interesting issues raised by the success of the $S^1/(Z_2 \times Z'_2)$ orbifold GUT models and its generalizations remain to be investigated.

Acknowledgments

We are grateful to Arthur Hebecker for discussions. J.M.R. thanks the Theory Group, LBNL, for their kind hospitality during this work.

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